SYNTHETIC SEISMOGRAMS OF THE HEAD WAVES
DUE TO THE WATER/SOLID ENVIRONMENT

Sıvı/Katı Ortamdan Kırılan Baş Dalgalarının
Yapay Sismogramları

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ABSTRACT
Beyond the critical angle of the total reflection head (lateral) waves may be generated in the upper medium (water). It has a peak amplitude at the critical angle. Recent literature includes the terms of absorption and shear wave velocity of the solid environment.

In this paper, synthetic seismograms of the head waves due to a known source are presented for different parameters of the medium including water and solid layer.

ÖZET
Baş dalgası kritik açının ötesinde birinci ortamda oluşabilir. Bu dalgının genişliği kritik açı değerinde en büyük olur. Son yıllarda katı ortama sağırma ve kesme dalgasi hızı terimleri eklenmiştir.

Bu makalede, bilinen bir kaynak kullanılarak baş dalgalarına ait yapay sismogramlar sıvı/katı tabakalı ortamanın değişik parametreleri için üretilmiştir.

INTRODUCTION
When the angle of incidence of a wave is equal to or greater than the critical angle, the so-called lateral wave may be observed in the upper medium. The region close to the critical point shows a special case as there would be a difficulty on separating the head wave from the reflected wave. Especially, if the compressional wave velocity in the solid layer is lower than the expected value, separating these two waves may not be possible. Actually, at the critical point head waves and the reflected waves join together. As a result of this, large amplitudes are observed in the vicinity of the critical point. Total wave field can be computed for this region.

Heelan (1953) developed the theory of head waves. Datta & Bhowmik (1969) investigated the head waves in two dimensional model. Mott (1971) worked out reflection and refraction coefficients at a fluid/solid interface. The innovation in Mott’s work was that the medium is assumed as lossy. Kumar and Raghava (1981) analyzed the amplitudes on reflection records of some of the shallow refraction profiles shot primarily for detailing the near-surface structure in granitic-terrain.

It has often been easy to estimate all other parameters such as absorption, longitudinal wave velocity and density of the bottom (Brekhovskikh 1980). It is more difficult to estimate the shear wave velocity of the solid layer. The synthetic seismograms of the lateral wave for the region close to the critical point may bring information about the shear wave velocity. The shear wave creates a disturbance along the water/solid interface and does not pass into the upper medium (water). However, the compressional wave which does travel in water comprises the influence of the shear wave of the solid layer (Godin 1983).

Considering a water/solid environment, the medium is assumed as inhomogeneous as upper and lower mediums with different velocities in between them (Fig. 1). Hence, It should be noted that water and the solid layer are individually assumed purely homogeneous. The physical parameters are denoted by p, Cw, in the upper medium as density and velocity of water and P1, CL, CT in the lower medium as density of the bottom, longitudinal and shear wave velocities, respectively.

Source and receiver are located in the upper medium (water). In Fig. 1, Zs and Z are the distances from the source and the receiver to the seabottom. θ is the angle of incidence, δ1 is the critical angle due to the longitudinal wave.

Fig. 1. The considered medium. Water thickness is chosen maximum 200 metres. The source and the receiver are located anywhere in water. The solid line shows the head wave path in water, seabed and again water. The dashed line is the reflected wave path.

Şekil 1. İnceленen ortam. Su tabakasının kalınlığı 200 metre. Alıcı ve vericinin konumları su içinde herhangi bir yerdedir. Kesikli çizgi baş dalgası yönü su, deniz yataşı ve tekrar su olarak göstermektedir.

Fig. 2. Typical performances of acoustic sources.

Şekil 2. Akustik kaynakların tipik çalışma aralıkları.
Synthetic Seismograms

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Application of the steepest-descent path method to the integral representation of the reflected wave field resulted in the limitations and these limitations are expanded by the parabolic cylinder functions (BrekhoVskikh 1980). However, a new expansion which is not restricted by the limitations for large k (wavenumber) and R_i (distance from the image of the source to the receiver) is introduced (Godin 1983). Therefore, the solution for the head wave (ψ_{li}) for the entire region of (R_i \rightarrow R_s) is given by Godin (1983) shown in equation (1).

\[ \psi_{li} = \psi_{li1} \left[ z (-dl)^{1/2} (d/dl)^{1/2} \right] \left[ D_{li2} \left( dl \exp (d^2/4) \right) \right] \]  

\[ \psi_{li1} = \frac{2i \sin \delta_i \exp [ikR_i \cos (\theta_i - \delta)]}{mkR_i [\sin \theta_i \cos \delta_i \sin \delta_i (\theta_i - \delta)]^{1/2}} \left( 1 - \frac{2}{k_i^2} \right) \]  

\[ D_{li2} \] is the parabolic cylinder function. (see Godin, 1983 for the choice of the suffix 1/2 and the properties of this function).

In equation (2), the first term is identical with the BrekhoVskikh's (1980) notation. When the lower half space has shear elasticity, the second term is included (Godin 1983).

\[ dl = \exp \left( \frac{3\pi}{4} \right) (kR_i)^{1/2} \sin \left[ \frac{(\theta_i - \delta_i)}{2} \right] \]  

\[ \delta_i \] is the critical angle due to the longitudinal wave.

\[ \delta_i = \arcsin \left( \frac{C_i}{C_s} \right) \]  

\[ k_i = \left( \frac{2\pi}{C_i} \right); \] \[ k_i = \left( \frac{2\pi}{C_s} \right); \] \[ C_i \] \[ C_s \] \[ m \] is the ratio of densities (p/p_i).

For small amount of absorption, \( \delta_i = \delta_i^* + i\delta_i^* \) may be defined as complex quantity and used in equation (4) instead of \( \delta_i \). In the following calculations of the synthetic seismograms, absorption will be added as imaginary in terms of Nepers/metre (Unick 1982).

**SEISMIC (ACOUSTIC) SOURCES AND CONSTRUCTING TIME DOMAIN SYNTHETIC SEISMOGRAMS**

The selection of the seismic source is often based upon requirements for resolution and depth of penetration. Hereby, a shallow seabed of 50 metres is considered. Water depth is as deep as continental shelf which is maximum 200 metres from the sea surface to the bottom of the sea. The acoustic source that will be discussed, listed in the order of decreasing resolution and increasing penetration are; the tuned transducer, the Unibooms (2 types) and the airgun (Fig. 2).

Tuned transducer system for the profiling is selected on the basis of their frequency range (generally 1.5 kHz to 7.5 kHz) and power available from the transmitter (12 KW to 500 KW). The high frequency systems provide good resolution but, are limited in penetration (Fig. 2).

To improve penetration, low frequency transducers are used with increasing amount of electrical power in order to provide high energy level. Some typical source performances are given in Fig. 2 (Geyer 1983).

The obvious desirable characteristics are for marine source that it must radiate a pressure wavefield of appropriate strength combined with a suitable signature and it must be repeatable with cycle time approaching the refrac tion record duration. In the following a special type of a seismic source (wavelet) will be used (Fig. 3a). This source function could conceivably be produced by a tuned transducer or an Uniboom type of transducer which is going to be used in the calculations of synthetic seismograms. The time duration of this special wavelet will be kept constant, as 16 msec. at all times. The mentioned wavelet is a real time signal and its amplitude spectrum is provided by using Fast Fourier Transform to the time domain seismic source function (Fig. 3b).

The time duration of such a wavelet might vary from a few tens of milliseconds to several hundred milliseconds. Therefore, the extent of the corresponding spatial waveform in the seabed may be anything from a few tens of meters or more, depending on the propagation velocity. Here, a single interface is considered and then all the process will be carried out as a single output with the identical phase for each head wave path coming from the seabed.

Synthetic seismograms are included in much of the literature as this has a significance on understanding the theoretical deductions for model structures. Most of the synthetic seismogram algorithms ignore the absorption. For not more than two decades, attention has been given to the problem of including absorption in the synthetic trace. However, most of these studies are based on reflection techniques and yet, there are few work have been done on refraction techniques on shallow marine seabed.

Calculating synthetic seismograms started with Peterson, et. al. (1955). Clearbou (1968) improved a computer programme for synthetic seismograms. Ganley (1981) has included the absorption to the medium and he also calculated the reflection and transmission coefficients for zero frequency.

The synthetic seismograms are calculated in the wavenumber-frequency domain and Fourier transformed back into time domain. The choice of the frequency domain as a working space is motivated by the fact that Godin's (1983) theory which is derived in the frequency domain from plane wave theory allows to include the effect
Fig. 3. a) Real time signal used as a time domain source function, b) The amplitude spectrum of the time domain signal shown in (a). Amplitude spectrum is provided using FFT and it shows the 0.48 dB of the amplitude spectrum of the source function.

Şekil 3. a) Zaman ortamı kaynağı fonksiyonu olarak gerçek zaman sinyali kullanılmıştır. b) (a) da görülen gerçek zaman sinyalinin genlik spektrumu. FFT kullanılarak elde edilen genlik spektrumunun yalnız 0.48 dB görüntülenmiştir.
Fig. 4. The steps which are involved in construction of the synthetic seismograms.

Şekil 4. Yapay sismogramların oluşturulmasını içeren basamaklar.
of attenuation.

In the frequency domain, synthetic seismogram equation to an impulse function is given by
\[ G(w) = H(w) u(w) \]  
(5)
where \( u(w) \) is the transform of the time domain source function of an impulse, \( H(w) \) is the transfer function and \( G(w) \) is the Fourier transform of the impulse response.

Knowing the response of the model to a spike input allows us to use a wavelet other than a spike. Godin’s (1983) equation for the lateral wave is the response to an input spike. Here, a source function other than a spike is used (Fig. 3a). The transfer function of the model is multiplied with the Fourier transform of the source function and the synthetic seismograms are in the frequency domain:
\[ S(w) = F(w) G(w), \]  
(6)
where \( F(w) \) is the Fourier transform of the real time domain function and \( S(w) \) is the frequency domain synthetic seismogram.

Fig. 4 shows the steps which are involved in construction of a synthetic seismogram.

Two different models have been chosen and the parameters of the models are given in table 1.

In order to obtain frequency domain synthetic seismograms first, Godin’s lateral wave equation (1) has been used to calculate amplitudes in terms of frequency. Thus impulse responses are in the frequency domain at fixed distances beyond the critical angle. An illustration of the medium is given in Fig. 5 in which H1, H2, H3, H4 show a hydrophone array.

In order to restore the frequency domain seismograms in the time domain, Inverse Fast Fourier Transform has been employed. The synthetic seismograms resulting from these simulations are constructed in two models. Table 1 shows the parameters of the models, model (i) and model (ii), varying in depth, absorption and shear wave velocity. Figures 6, a, b, c and 7a, b, c show the synthetic seismograms resulting from these models (model (i) and model (ii), respectively), which are the source responses of the seabed transmission filters.

CONCLUSIONS

As it is seen from the graphical results that the transmitted signatures are delayed relative to each other. To determine the magnitude of the each delayed transmitted signature may be compared to the source pulse used to generate it. Complex phase lag with the contribution of absorption has led to shape changes for the received pulses as each of the received pulse has an identical phase at the boundary. Presence of shear wave in the solid environment

<table>
<thead>
<tr>
<th>Parameters of the model</th>
<th>Model (i)</th>
<th>Model (ii)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Seismograms presented in Figure</td>
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<td>7</td>
</tr>
<tr>
<td>Depth (m)</td>
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<td>25</td>
</tr>
<tr>
<td>Density (kg/m³)</td>
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<td>solid</td>
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<td></td>
<td>1000</td>
<td>2700</td>
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<tr>
<td>Compressional wave velocity (m/s)</td>
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<td>2400</td>
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<tr>
<td>Absorption (Nepers/m)</td>
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<td>a b c</td>
</tr>
<tr>
<td></td>
<td>0 0.14 0.14</td>
<td>0 0.14 0.14</td>
</tr>
<tr>
<td>Shear wave velocity (m/s)</td>
<td>640</td>
<td>640</td>
</tr>
</tbody>
</table>
Fig. 6. Synthetic seismograms for model "i". a) No absorption and shear wave velocity are included in the synthetic traces. b) Absorption (0.14 Nepers/m) has been included in the synthetic traces shown in (a). c) Shear wave velocity (640 m/s) has been included in the synthetic traces shown in (b).

Şekil 6. Model "i" için yapay sismogramlar. a) Yapay ızlere soğurma ve kesme dalga hızı eklenmemiştir. b) (a) da görülen yapay 0.14 Nepers/m düzeyinde soğurma eklenmiştir. c) (b) de görülen ız 640 m/s düzeyinde kesme dalgası hızı eklenmiştir.

Fig. 7. Synthetic seismograms for the model "ii". a) No absorption and shear wave velocity are included in the synthetic traces. b) Absorption (0.14 Nepers/m) has been included in synthetic traces shown in (a). c) Shear wave velocity (640 m/s) has been included in the synthetic traces shown in (b).

Şekil 7. Model "ii" için yapay sismogramlar. a) Yapay ızlere soğurma ve kesme dalga hızı eklenmemiştir. b) (a) da görülen yapay 0.14 Nepers/m düzeyinde soğurma eklenmiştir. c) (b) de görülen yapay ız 640 m/s düzeyinde kesme dalgası hızı eklenmiştir.
resulted in small increases in the amplitudes of the synthetic traces.

Consequently, in Figures 6a, b, c and 7a, b, c transmissions through the synthetic seabed models are shown. The synthetic traces in Figures 6a and 7a do not include absorption and shear wave velocity terms of the lower medium. Figures 6b and 7b show when there is absorption of 0.14 nepers/meter and Figures 6c and 7c show when shear wave velocity level increased to 640 m/s. in the lower medium. The synthetic seismograms are sensitive to change in absorption and shear wave velocity and therefore head wave may carry significant information about the rigidity of the seabed from the seabed.

REFERENCES
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Urick, R.J. 1982, Sound propagation in the Sea, Peninsula pub., California.