Performance of the finite element method for regional - residual separation on gravity method

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Abstract: Separation of regional and residual anomalies in potential field applications has been studied considerably for years. Computing regional anomaly is a critical step in modeling and inversion in the gravity method. A number of techniques, both in space and frequency domains, have been developed for regional-residual resolution. Finite element approach is relatively a recent and new technique to compute the regional component. In this paper, data processing techniques, such as trend analysis, filtering, and finite element method were applied on synthetic and field gravity data to separate regional and residual gravity anomalies. In the synthetic applications, three structurally different models were used. Model I consists of three blocks having different volumes, geometrical shapes and density contrasts. Model II and Model III consist of two and three cubic blocks, respectively, having different volume and the same density contrast. In the real data application, the field gravity data were observed on the well-known Aswaraopet Fault (India). Basement depth of the Chintalpudi basin deduced from borehole information is approximately 3 km. The obtained results for all applications were compared with the ones from conventional data analysis methods such as the filtering and trend analysis. As a result, the Finite Elements Method is more preferable with respect to the conventional ones.

Key words: Synthetic data, gravity, the finite element method, regional anomaly, residual anomaly

INTRODUCTION

Realizing the importance of the regional-residual separation, attempts have been made repeatedly during the past decades in order to improve the techniques and algorithms for accurate computation of the regional and residual fields. Trend surface analysis, polynomial fitting and a variety of filtering schemes are a few standard techniques to compute the residual anomalies. However, these approaches are often inadequate. Contrarily, the regional field has been successfully computed by finite element approach. The present study provides a comparative performance of trend surface and filtering techniques vis-a-vis finite element technique.

Although, there are improvements in the existing methods and algorithms to separate regional and residual gravity anomalies, there is no significant difference between polynomial fitting and filtering techniques. High degree trend analysis and filtering techniques are more effective using high speed computing power. Recently different techniques such as Finite Elements (Mallick and Sharma, 1999; Kaftan, 2003 and Kaftan et.al, 2005), wavelet transform and spectrum analysis (Fedi and Quarta, 1998; Xu et.al, 2009), have been used to eliminate residual anomaly effects from computed regional anomalies. In this paper, synthetic gravity data are first generated in order to study the performance of the Finite Elements Method (FEM) and the conventional data analysis methods. Random noise (1 and 5%) is added to the synthetic data. Finally, these methods are applied on the field gravity data from Aswaraopet fault (India) (Chakravarthi, 2009).

REGIONAL FIELD CALCULATION BY THE FINITE ELEMENTS METHOD (FEM)

The Finite Elements Method (FEM) is applied by dividing a survey area into sub-segments. The elements are usually isoparametric and the nodes on these elements are defined (Mallick and Sharma, 1999). The characteristics of isoparametric finite elements could be described by the same level and the same interpolation functions for every point position and displacement. An eight-node rectangular iso-parametric element type was used and the gravity values of these nodes were computed by placing the rectangular element over the precomputed anomaly map to compute the regional gravity anomaly. This method can be
easily applied to any scale anomaly map; therefore, it is more useful than the other methods, such as trend analysis (Davis, 1986) and filtering (Fuller, 1967).

Observed gravity data describe the total effect of both deep and shallow structures (Pawlowski, 1994): 
\[ g(x,y) = g_s(x,y) + g_d(x,y) \]  
where, \( g(x,y) \) are the observed gravity data, \( g_s(x,y) \) is the gravity effect of shallow structures, and \( g_d(x,y) \) is the gravity effect of deep structures. Apart from the conventional data analysis methods, FEM is also used for the separation of the regional and the residual anomalies. This method has advantages over the conventional data analysis methods because only a few observation points on a Bouguer gravity map are needed to compute the regional gravity anomaly.

The regional anomaly is computed (Mallick and Sharma, 1999; Sarma et al., 1993) by: 
\[ g_r(x,y) = \sum_{i=1}^{8} N_i(x,y) g_i \] 
where \( g_r(x,y) \) represents the regional field, \( N_i(x,y) \) represents the shape functions, and \( g_i \) represents the gravity values at the nodes. An eight node quadrilateral element in the \((x,y)\) plane is shown in Fig. 1, where the real map space \((x_c,y_c)\) represents the center of this element. The length of the sides is \(2a\) and \(2b\), respectively. Changing the existing \((x,y)\) coordinates to \((\xi,\eta)\) coordinates, as illustrated in Fig. 2, is a necessary step.

The new \((\xi,\eta)\) coordinates can be expressed (Mallick and Sharma, 1999) as:
\[ \xi = \frac{x - x_c}{a}, \quad \eta = \frac{y - y_c}{b} \] 

The shape functions for the nodes in Fig. 2 can be expressed (Cheung and Yeo, 1979) as:
\[ N_i(\xi,\eta) = \frac{(1+\xi \xi_i)(1+\eta \eta_i)(\xi \xi_i + \eta \eta_i - 1)}{4} \] 
for \( i = 1, 3, 5 \) and \(7\) only
\[ N_i(\xi,\eta) = \frac{(1-\xi^2)(1+\eta \eta_i)}{2} \] 
for \( i = 2 \) and \(6\) only
\[ N_i(\xi,\eta) = \frac{(1+\xi \xi_i)(1-\eta^2)}{2} \] 
for \( i = 4 \) and \(8\) only.

It should be noted that \( N_i(\xi,\eta) = 1 \) at the \(i\)th node and zero at the other nodes, therefore
\[ \sum_{i=1}^{8} N_i(\xi,\eta) = 1 \] \(4d\)

The regional gravity is expressed in terms of the shape functions (Mallick and Sharma, 1999) as:
\[ g(\xi,\eta) = \sum_{i=1}^{8} N_i(\xi,\eta) g_i \] \(5\)

Based on the new reference plane, the new coordinates could be given (Mallick and Sharma, 1999) as:
\[ x(\xi,\eta) = \sum_{i=1}^{8} M_i(\xi,\eta) x_i \] \(6a\)
\[ y(\xi,\eta) = \sum_{i=1}^{8} M_i(\xi,\eta) y_i \] \(6b\)

where \( M_i(\xi,\eta) \) represents the shape functions, \( x_i \) and \( y_i \) represent the new node coordinates.

These equations provide the transformation of the computed regional anomaly from the reference \((\xi,\eta)\) plane to the real \((x,y)\) plane. In the equations (5), (6a) and (6b), the coordinates of any given point and field variable use the same shape functions, that is, \( N_i(\xi,\eta) = M_i(\xi,\eta) \), called isoparametric elements. This is one of the important features of the isoparametric elements.
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FIG. 2. Eight-node quadrilateral element in the nondimensional $\xi$-$\eta$ plane (Mallick and Sharma, 1999).

APPLICATIONS

This filter technique was tested on synthetic and field gravity data and compared with conventional methods such as filtering and trend analysis. The observed gravity data contain noise. For this reason FEM and the conventional methods were applied to noisy synthetic data to test the performance. Gaussian noise 1% and 5% was added to the synthetic gravity data. FEM was also applied to field data observed on the listric fault in India.

Model I

In the first scenario, the model consists of three blocks (two cubic and a quadrilateral) having different volume, depth, and density contrast. The synthetic anomaly data were calculated using the formula described in Grant and West (1965) (See Appendix A). As shown in Figure 3, the first cubic block is located at depth of 5 km, the quadrilateral block, at depth of 15 km and the second cubic block, at depth 50 km. The symbols $2a$, $2b$ and $2c$ in Figure 3 indicate the width of the block in the $x$-direction, the length of the block in the $y$-direction and the thickness of the block in the $z$-direction, respectively. The block size and the density contrast are denoted in Figure 3. The gravity of the deeper cubic block corresponds to the regional effect, while the gravity of the shallow cubic and quadrilateral bodies to the residual. The regional and residual anomalies were separated by filtering techniques, trend analysis, and finite element methods (Figure 4a - g). The corresponding gravity profiles along A-A’ are also displayed in Figure 4h and 4k.

FIG. 3. Model I.
FIG. 4.
(a) Synthetic gravity anomaly map of Model I.
(b) Low-pass filtering (cut-off frequency: 0.1 cycle/grid spacing).
(c) Third order regional trend analysis.
(d) Regional anomaly derived by FEM.
(e) High-pass filtering (cut-off frequency: 0.1 cycle/grid spacing).
(f) Third order residual trend analysis.
(g) The residual anomaly derived by FEM.
(h) Regional profiles along A-A' obtained by trend analysis, low-pass filtering and FEM.
(k) Residual profiles along A-A' obtained by trend analysis, high-pass filtering and FEM.
The gravity field data contain noise. For this reason, the performance of FEM was also tested by adding Gaussian noise of 1% and 5% to the previously presented noise-free synthetic gravity data. Synthetic gravity anomaly maps with noise are illustrated in Figure 5 and Figure 6. In the same figures (Figs. 5b, 5c, 6b and 6c) the gravity profiles are presented along A-A’. Root Mean Square Errors (RMSE) are given on Table 1. Even for 5% noise level, FEM results have lower RMSE value for both regional and residual anomalies.

![Synthetic Gravity Anomaly Map](image1)

**FIG. 5.** (a) Synthetic gravity anomaly map with 1% level Gaussian additive noise. (b) Regional profiles along A-A’ obtained by trend analysis, low-pass filtering and FEM. (c) Residual profiles along A-A’ obtained by trend analysis, high-pass filtering and FEM.

![Gravity Profiles](image2)

**FIG. 6.** (a) Synthetic gravity anomaly map with 5% level Gaussian additive noise. (b) Regional profiles along A-A’ obtained by trend analysis, low-pass filtering and FEM. (c) Residual profiles along A-A’ obtained by trend analysis, high-pass filtering and FEM.

**TABLE 1.** RMSE values of for Model I.

<table>
<thead>
<tr>
<th>Noise Level (%)</th>
<th>0</th>
<th>1</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Regional Anomalies RMSE</strong></td>
<td><strong>Low-pass Filter</strong></td>
<td>9.6</td>
<td>9.8</td>
</tr>
<tr>
<td></td>
<td><strong>Trend Analysis</strong></td>
<td>6.8</td>
<td>6.8</td>
</tr>
<tr>
<td></td>
<td><strong>FEM</strong></td>
<td>1.8</td>
<td>1.9</td>
</tr>
<tr>
<td><strong>Residual Anomalies RMSE</strong></td>
<td><strong>High-pass Filter</strong></td>
<td>9.5</td>
<td>9.5</td>
</tr>
<tr>
<td></td>
<td><strong>Trend Analysis</strong></td>
<td>3.7</td>
<td>3.8</td>
</tr>
<tr>
<td></td>
<td><strong>FEM</strong></td>
<td>1.8</td>
<td>1.9</td>
</tr>
</tbody>
</table>
Model II

In this model, two cubic blocks having different volume and depth but the same density contrast were used to compute synthetic gravity anomaly. As shown in Figure 7, the first block is located at depth of 5 km (residual field) and the second block, at depth of 20 km (regional field) under the first block. First and second cubic block sizes are 6 km and 12 km, respectively. The regional and residual anomalies were computed by filtering techniques, trend analysis, and FEM and the results are given in Figure 8a - 8g. The regional and residual profiles along B-B’ from the conventional methods and FEM are shown in Figure 8h and 8k. As in the previous synthetic dataset, the additive Gaussian noise is 1% and 5%. The anomaly maps with the noise are shown in Figure 9a and Figure 10a. The corresponding profiles along B-B’ are displayed in Figure 9b-c and Figure 10b-c. According to RMSE (Table 2), the FEM reproduces more accurately the regional and the residual anomalies, even for the anomalies containing 5% Gaussian random noise.

Model III

In the last application on the synthetic data, the model consists of three cubic blocks which have the same density contrast although they have different volumes, locations and depths. The purpose of this application is to obtain the residual effect (two shallow structures). As shown in Figure 11, the first cubic block is located at depth of 1 km, the second cubic block, at 2 km and the third cubic block, at 3 km. The length of the cubic blocks in x-y and z directions are 400, 600 and 800 m, respectively.

All cubic blocks located at 1, 2 and 3 km depth for the residual effect. Filtering techniques, trend analysis, and FEM were applied to the synthetic gravity data (Fig. 12a-d. The FEM separated the residual anomalies more effectively even if they were located closer each other. The residual profiles along C-C’ from these methods are shown in Figure 12e. Table 3 summarizes the RMSE values.

Field Data

The FEM and conventional methods were applied to field gravity data from Chintalpudi subbasin (India). The basin is bounded by Aswaraopet fault to the east. The length of the fault is over 20 km and strikes NNW-SSE. According to borehole data in the basin, the depth of basement was estimated 2935 km. The gravity map of the region is shown in Figure 13a (Chakravarthi, 2009). The regional and residual field are shown in Figure 13b-d and 13e-g, respectively. Gravity profiles along A-B (Figure 13h and Figure 13k) indicate that the FEM is more preferable.
FIG. 8.
(a) Synthetic gravity anomaly map of Model II.
(b) Low-pass filtering (cut-off frequency: 0.1 cycle/grid spacing).
(c) Third order regional trend analysis.
(d) Regional anomaly derived by FEM.
(e) High-pass filtering (cut-off frequency: 0.1 cycle/grid spacing).
(f) Third order residual trend analysis.
(g) The residual anomaly derived by FEM.
(h) Regional profiles along B-B’ obtained by trend analysis, low-pass filtering and FEM.
(k) Residual profiles along B-B’ obtained by trend analysis, high-pass filtering and FEM.

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FIG. 9. (a) Synthetic gravity anomaly map of Model II with 1% Gaussian additive noise. (b) Regional profiles along B-B’ obtained by trend analysis, low-pass filtering and FEM. (c) Residual profiles along B-B’ obtained by trend analysis, high-pass filtering and FEM.

TABLE 2. RMSE values for Model II.

<table>
<thead>
<tr>
<th>Noise Level (%)</th>
<th>REGIONAL ANOMALIES RMSE</th>
<th>RESIDUAL ANOMALIES RMSE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Low-pass Filter</td>
<td>Trend Analysis</td>
</tr>
<tr>
<td>0</td>
<td>7.4</td>
<td>4.2</td>
</tr>
<tr>
<td>1</td>
<td>7.4</td>
<td>4.2</td>
</tr>
<tr>
<td>5</td>
<td>7.5</td>
<td>4.4</td>
</tr>
</tbody>
</table>
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FIG. 11. Model III.

TABLE 3. RMSE values for Model III.

<table>
<thead>
<tr>
<th>NOISE FREE RESIDUAL ANOMALIES</th>
<th>RMSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>High-pass Filtering</td>
<td>Trend Analysis</td>
</tr>
<tr>
<td>0.16</td>
<td>0.21</td>
</tr>
</tbody>
</table>

FIG. 12.
(a) Synthetic gravity anomaly map of Model III.
(b) High-pass filtering (cut-off frequency: 0.1 cycle/grid spacing).
(c) Third order residual trend analysis.
(d) Residual anomaly derived by FEM.
(e) Residual profiles along C-C' obtained by trend analysis, high-pass filtering and FEM.
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**FIG. 13.**
(a) Observed gravity anomaly map of the Chintalpudi basin (Chakravarthi, 2009).
(b) Low-pass filtering (cut-off frequency: 0.1 cycle/grid spacing).
(c) Third order regional trend analysis.
(d) Regional anomaly derived by FEM.
(e) High-pass filtering (cut-off frequency: 0.1 cycle/grid spacing).
(f) Third order residual trend analysis.
(g) The residual anomaly derived by FEM.
(h) Regional profiles along A-B obtained by trend analysis, low-pass filtering and FEM.
(k) Residual profiles along A-B obtained by trend analysis, high-pass filtering and FEM.
RESULTS AND CONCLUSIONS

The validity of FEM is tested on synthetic data with and without random noise as well as on field gravity data. Filtering techniques, trend analysis, and FEM were applied on synthetic gravity anomalies computed for bodies having different depth, geometry and density contrast. It was shown that regional effects caused by deeper bodies could be better separated with the FEM, according to RMSE values. The field gravity data from Aswaraopet fault in Chintalpudi subbasin (India) were interpreted by FEM and the conventional methods. The residual gravity anomaly of the shallow structure (basin) is separated well from the regional one using FEM.

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Synthetic Gravity Anomaly Computation

Cubic and quadrilateral blocks (Fig. A1) having different volume and density contrast were used to compute theoretical gravity anomaly. 3-D gravity anomaly of such blocks is given by the equation:

$$\Delta g(x, y) = \frac{B_0}{R^3} z_0 + 3B_2^0 \left[ (3\cos^2\psi - 1)z_0^3 + (5\sin^2\psi \cdot \sin^2\psi - 2\cos^2\psi - 1)x^2 z_0 \right.$$  

$$+ (5\cos^2\psi \cdot \sin^2\psi - 2\cos^2\psi - 1)y^2 z_0 - 2\sin\psi \cdot \sin\psi \cdot \cos\psi (x^3 + xy^2 - 4xz_0^2) \right.$$  

$$+ 2\cos\psi \cdot \sin\psi \cdot \cos\psi (y^3 + x^2y - 4yz_0^2) - 10\sin\psi \cdot \cos\psi \cdot \sin^2\psi (xyz_0) \right] / 2R$$  

$$+ B_2^0 \left[ -3\sin^2\psi z_0^3 + (5\cos^2\psi - 5\sin^2\psi \cdot \cos^2\psi + 2\sin^2\psi) x^2 z_0 \right.$$  

$$+ (5\sin^2\psi - 5\cos^2\psi - 2\sin^2\psi) y^2 z_0 + 2\cos\psi \cdot \sin\psi \cdot \cos\psi (y^3 + x^2y - 4yz_0^2) \right.$$  

$$- 2\sin\psi \cdot \sin\psi \cdot \cos\psi (x^3 + xy^2 - 4xz_0^2) + 10\sin\psi \cdot \cos\psi (1 + \cos^2\psi) xyz_0 \right] / R^7$$

where

$$R = (x^2 + y^2 + z_0^2)^{1/2}$$

$$B_0^0 = 8 \text{ Gpabc}$$

$$B_2^0 = \frac{B_0^0 (2c^2 - a^2 - b^2)}{6}$$

$$B_2^2 = \frac{B_0^0 (a^2 - b^2)}{12}$$

and $a$, $b$ and $c$ is the half of the body size in $x'$, $y'$ and $z'$ -direction, respectively (Fig. A2), $\rho$ is density contrast, and $z_0$ is the depth of block center.